Universität zu Köln

Mathematisch- Naturwissenschaftliche Fakultät
Seminar für Mathematik und ihre Didaktik

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Complex Dimensions of Cantor Strings and the Riemann Zeros

Abstract:

A fractal string (one-dimensional fractal drum) is a sequence $L : l_1, l_2, l_3, \ldots$ that decreases to zero. Its zeta function is $\zeta_L(s) = \sum_{n=1}^{\infty} l_n^s$. The sound of L, its frequencies, is described with the aid of the Riemann zeta function, $\zeta(s) = 1 + 2^{-s} + 3^{-s} + \ldots$, using the idea of complex dimensions of L. The location of the zeros of $\zeta(s)$ turns out to be crucial to describe its sound. I will explain how the functional equation for $\zeta(s)$ could have been discovered by Riemann after studying Euler's work. On the special class of Cantor strings, one can study the sound by a direct computation. The conclusion for the Riemann zeta function is that its zeros do not lie in a vertical arithmetic progression. This talk is interesting for everybody interested in Number Theory, History of Mathematics, and Fractal Geometry.